

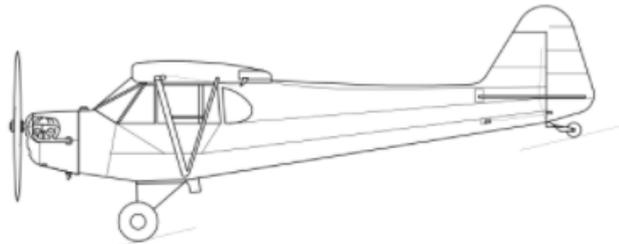
3 Concepts and Computational Models in Wing Design

- 3.1 Introduction - Mapping Planform to Lift and Drag
- 3.2 Computational Vortex-Lattice Methods
- 3.3 Planform Design Studies with VLM
- 3.4 Wings for High Speed

Tutorial: --

Review questions

1. Look at the Biot-Savart equation (3.5). Where does it produce infinite velocities? Consider now a VLM model with a symmetric main wing ahead of a vertical tail in the symmetry plane and a horizontal stabilizer. The model assumes that the trailing vortices follow free stream streamlines. Consider the Piper Cub J-3 in the sketch below, from Wikipedia. Sketch in the angles of attack where we expect trouble from trailing vortices hitting collocation points or bound vortex midpoints. Yet the Cub flies just fine. Discuss the effect of the fuselage. In zero sideslip, the wing horseshoe trailing leg in the symmetry plane should not create problems in the force calculation anyway. Why?



The VLM model is unphysical in such cases of close encounters. Yet it is prudent to check real flow fields to see where in the flight envelope forebody separated vortices hit aft located lifting surfaces. The Super Hornet F-18 has wing leading edge strakes producing strong vortices that hit the V-tail, shake it, and reduce its fatigue life.

2. Assume that a pair of wingtip vortices Γ and $-\Gamma$, separated by wing span b (actually, they will converge somewhat from the wing tips) are infinitely long and horizontal. How fast do they sink – or rise? Using the simplest constant-strength Prandtl lifting line model, compute Γ for a Boeing 747 at take-off weight around 350 metric tons. You will need more data, easily available. Constant strength is wrong; re-do the calculation assuming elliptic lift distribution; that model sheds a variable-strength wake. Assume that it rolls up so all vorticity from the right half is concentrated to the wing tip trailing streamline. What is its strength?

VLM computations

The script `tor1.m` computes a quadrilateral wing defined by AR , taper ratio ct/cr , quarter-chord sweep angle $Sweep$, dihedral $Dihed$, and $Twist$ for some airspeed, altitude, and angle of attack α . It produces a row in a table of the parameters (angles in radians)

`AR ct/cr Sweep Dihed. Twist alpha`

and a corresponding row in a table of the force and moment coefficients

`CL CD CY Cl Cm Cn`

`tor1(AR, tap, swe, dih, twi, alt, airspeed, alpha, foil1, foil2, resimn, resimx)`

`resimn` and `resimx` define refinements of the initial paneling:

$$n_x = 2^{(res-1)} n_{x0} \text{ for } res = resimn \dots resimx.$$

so `resimn = 2` and `resimn = 4` runs original, doubled, and quadrupled paneling used to extrapolate the results, in `VLM_e.m` described below.

3. Check how the VLM estimates for C_L , C_m converge with the number of vortices. Run with script `tor1.m` an AR 8 rectangular wing with symmetric profile and

$$Nx = 2^k, k = 1, \dots \text{ and } Ny = 4Nx, \alpha = 5 \text{ deg.}$$

Use `resimn = resimx = 2, 3, 4, 5`. Plot C_L , C_m and C_D vs. $1/Nx$. Theory shows that

$$C_L(1/Nx) = C_L(0) + C1/Nx + \text{smaller terms.}$$

This allows extrapolation to $Nx = \text{Infy}$:

$$C_L(0) = 2 C_L(1/(2Nx)) - C_L(1/Nx)$$

is usually a better approximation than even $C_L(1/(4Nx))$ and much cheaper since the work grows like Nx^4 . The extrapolation can be continued with results from 4 Nx , 8 Nx , ... (but 8 Nx costs 4000 x the compute time of Nx). Systematic mesh refinement with at least three levels is a due diligence check that the convergence follows the theory so bad results from under-resolution, ill-conditioning, or "close encounters" can be spotted. The function `VLM_e.m` performs calculations of force and moment coefficients for a sequence of refinements of a user-defined initial coarse paneling.

4. Then try a straight wing with AR 2, 4, 6, 8, 10. Run an alpha sweep with `tor1.m` and check $dC_L/d\alpha$ (small α) vs. the formula in Ch 3. The script `ARsweep.m` runs a list of AR and α using `tor1.m` which in turn uses `VLM_e.m` and produces tables `partab` and `restab` with rows described above. Your job is to compute $C_{L,\alpha}$ from the table.
5. The script `tor1.m` computes a quadrilateral wing with sweep, taper, and twist. Your job is to experiment with sweep *and* taper for an untwisted wing. Modify `ARsweep.m` to make an (AR, taper, α) sweep and fit

$$C_{Di} = C_{D0i} + k C_L^2$$

to the computed points for each (AR,taper). The wing (induced drag) efficiency factor is

$e = k/(\pi AR)$ where lifting line theory predicts that $e_{max} = 1$ for an elliptical flat wing.

Fix AR at 6. Then run a quadrilateral tapered wing with $ct/cr = [0.3, 0.7, 1]$ and quarter-chord sweeps $[0,30,60]^\circ$. Note the e and C_{D0i} so computed as well as $C_{L,\alpha}$. Before analyzing the data, add a rough estimate of $C_{D0,viscous}$ of 150 cts to C_{D0i} . Show that the best L/D (which determines best glide slope) is $LoD_{max} = 1/\text{sqrt}(k C_{D0tot})$. Which wing has best glide slope?

6. The file `GAV.m` has a Tornado model for a Piper Cub-like generic general aviation airplane and its weight and CG-location. What angle of attack and elevator angle will trim it for straight and level flight at sea level and TAS 35 m/s? Suppose $C_{D0v} = 0.0150$. What thrust is required? First, neglect the elevator influence on lift to find α , then for this α find the correct elevator angle ele . A more accurate procedure is to assume "linear aerodynamics". Compute the derivatives $C_{L,\alpha}$, $C_{m,\alpha}$, $C_{L,ele}$ and $C_{m,ele}$ from a computed table produced by (α,ele) -sweep and solve the linear system

$$qS(0 + \alpha C_{L,\alpha} + ele C_{L,ele}) = W,$$

$$qScMAC (0 + \alpha C_{m,\alpha} + ele C_{m,ele}) = 0$$